Scheduling of oil-refinery operations

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Ignacio Grossmann[1]

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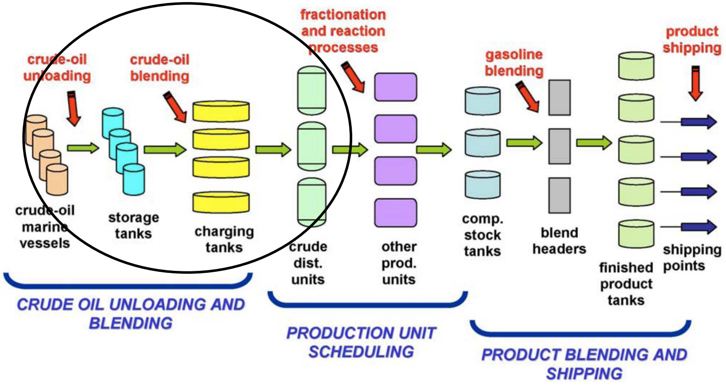
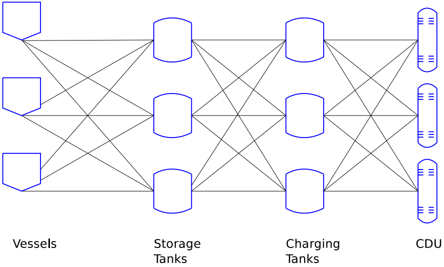


Méndez, C.A., Grossmann, I.E., Harjunkoski, I., Kaboré, P. A simultaneous optimization approach for off-line blending and scheduling of oil-refinery operations. Computers & Chemical Engineering, 2006, 30 (4): 614-634.

Crude oil unloading and blending: current example



- Non-simultaneous load and unload
- Vessels can unload to any storage tank.
- Bounds for blend composition in Charging tanks.
- Concentration limits are specified for the products.
- Product yields are specified for each crude.
- Minimum product yields for CDU.

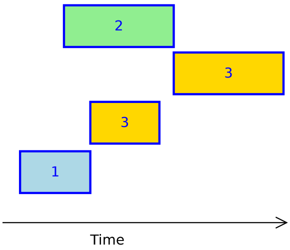
Priority-slot based formulation

A scheduling problem has two main decisions on time arrangement:

- Sequence of operations.
- Timing for each operation (start, duration, end).

We use **Priority-slot** based formulation ($\neq$ Time-slot) for **continuous-time** scheduling.
Separated variables for:

- Priority (Z).
- Timing (S, D, E).



Mouret, S., Grossmann, I., Pestiaux, P. A Novel Priority-Slot Based Continuous-Time Formulation for Crude-Oil Scheduling Problems. Industrial & Engineering Chemistry Research, 2009, 48 (18): 8515-8528.

MINLP Model

- Binary variables: Z_{iv}
  - Assignment decisions: Z_{iv}
- Continuous variables:
  - Time decisions: S_{iv}, D_{iv}, E_{iv}
  - Volume decisions: V_{iv}^t, V_{ivc}
  - Level decisions: L_{ir}^t, L_{irc}
- Linear constraints:
  - Continuous distillation: $\sum_{i\in T}\sum_{v\in I_r} D_{iv} = H$
  - Volume bounds: $\underline{V_v^t} \cdot Z_{iv} \leq V_{iv}^t \leq \overline{V_v^t} \cdot Z_{iv}$
  - Volume composition: $V_{iv}^t = \sum_{c\in C} V_{ivc}$
  - Level definition: $L_{irc} = L_{0rc} + \sum_{j\in T, j<i}\sum_{v\in I_r} V_{ivc} - \sum_{j\in T, j<i}\sum_{v\in O_r} V_{ivc}$
  - Level limitations: $\underline{L_r^t} \leq L_{ir}^t \leq \overline{L_r^t}$
  - Flowrate limitations: $\underline{FR_v} \cdot D_{iv} \leq V_{iv}^t \leq \overline{FR_v} \cdot D_{iv}$
  - Property specifications: $\underline{x_{vk}} \cdot V_{iv}^t \leq \sum_{c\in C} x_{ck} V_{ivc} \leq \overline{x_{vk}} \cdot V_{iv}^t$
  - Crude blend demands: $\underline{D_r} \leq \sum_{i\in T}\sum_{v\in O_r} V_{iv}^t \leq \overline{D_r}$
  - ...
- Nonlinear constraint:
  - Composition constraint: $\frac{V_{ivc}}{V_{iv}^t} = \frac{L_{irc}}{L_{ir}^t} \Leftrightarrow V_{ivc} \cdot L_{ir}^t = L_{irc} \cdot V_{iv}^t$

Mouret, S. Optimal Scheduling of Refinery Crude-Oil Operations. Ph.D. Thesis. Department of Chemical Engineering. Carnegie Mellon University. 2010.

Solution strategy

- Solution with global solver (BARON) for MINLP.

- Solution with two stages: MILP (CPLEX) + NLP (BARON or CONOPT). Loop: 20 cycles.

MILP Scheduling Algorithm

maximize
objective
s.t.
all constraints
except composition constraints

Fix assignment variables Z_{iv}

NLP

maximize
objective
s.t.
all constraints

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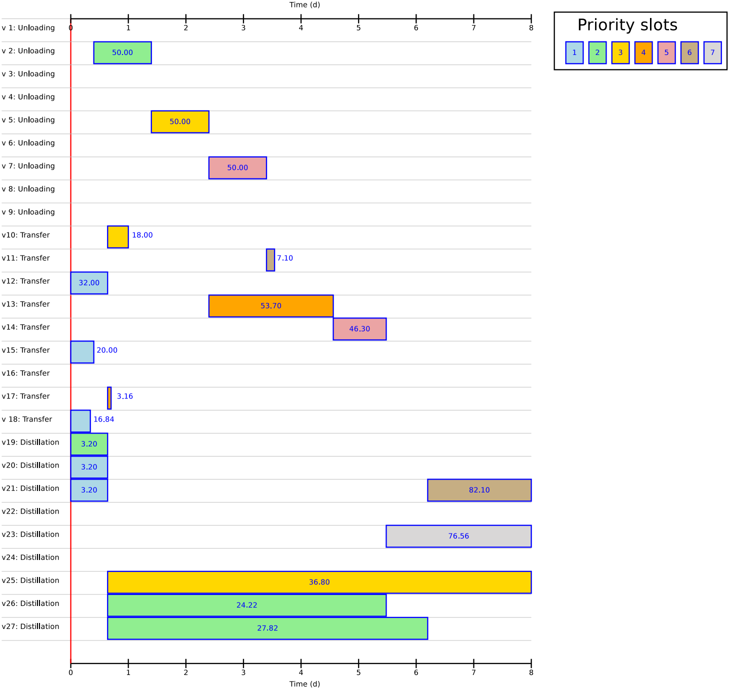
Solution

- Time horizon: 8 days.
- 7 Priority Slots.
  - 6 $\rightarrow$ the problem is not feasible.
  - 8 $\rightarrow$ the problem is larger.

	Global	
# Continuous Vars.:	2311	
# Discrete Vars.:	189	
# Constraints:	5738	
	1st stage (MILP)	**2nd stage (NLP)**
# Removed Constraints:	755	1043
# Constraints:	4983	4695

			Best solution		
Strategy	First stage	Second stage	CPU Time [s]	Obj. Fun.	Difference with best
MILP + NLP	CPLEX	BARON	3829	60.400	0.71%
MILP + NLP	CPLEX	CONOPT	159	60.829	0.00%
MINLP	BARON		(*) 90000	60.793	0.06%

(*) Time limit.



Time (d)
Priority slots
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v 1: Unloading
v 2: Unloading
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v 3: Unloading
v 4: Unloading
v 5: Unloading
50.00
v 6: Unloading
v 7: Unloading
50.00
v 8: Unloading
v 9: Unloading
v10: Transfer
18.00
v11: Transfer
7.10
v12: Transfer
32.00
v13: Transfer
53.70
v14: Transfer
46.30
v15: Transfer
20.00
v16: Transfer
v17: Transfer
3.16
v 18: Transfer
16.84
v19: Distillation
3.20
v20: Distillation
3.20
v21: Distillation
3.20
82.10
v22: Distillation
v23: Distillation
76.56
v24: Distillation
v25: Distillation
36.80
v26: Distillation
24.22
v27: Distillation
27.82
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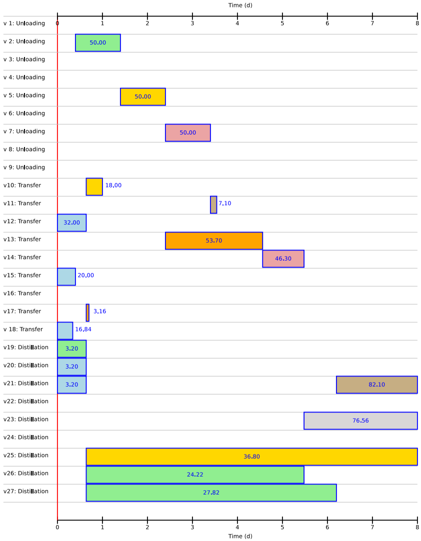
Time (d)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v 5: Unloading
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v 6: Unloading
v 7: Unloading
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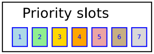
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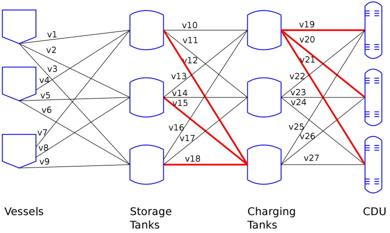
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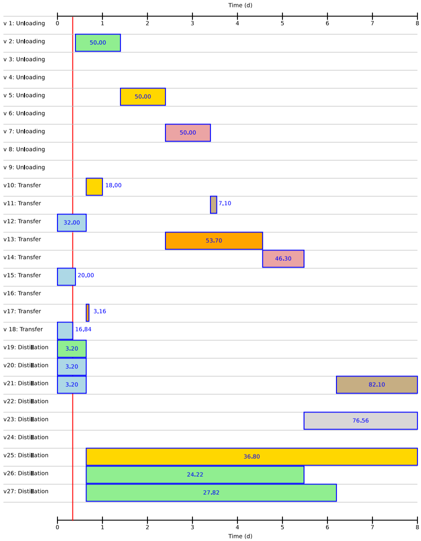
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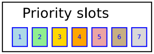
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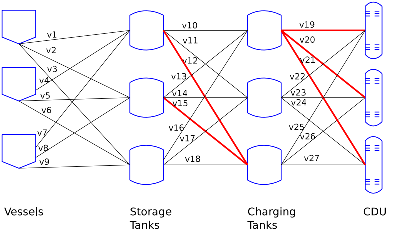
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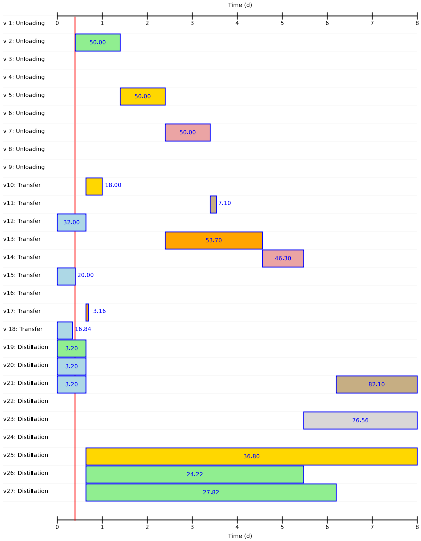
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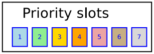
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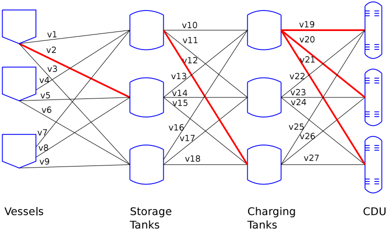
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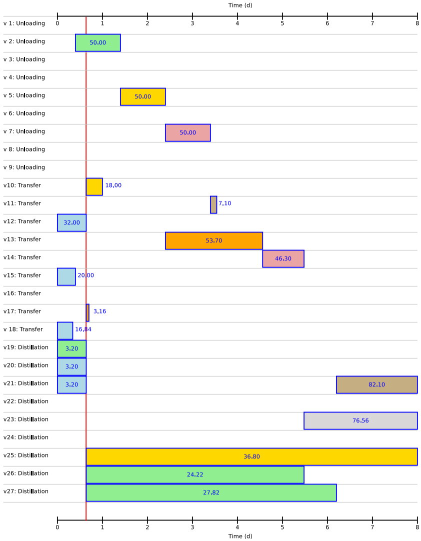
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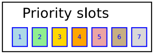
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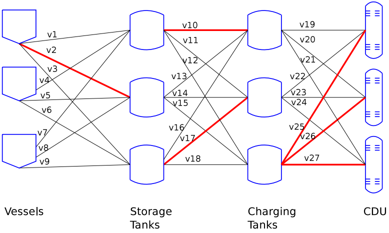
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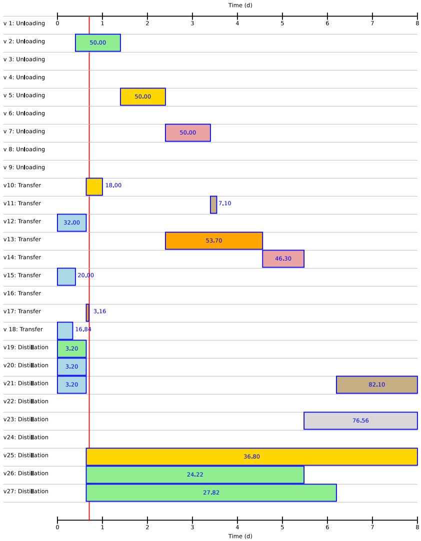
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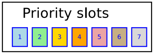
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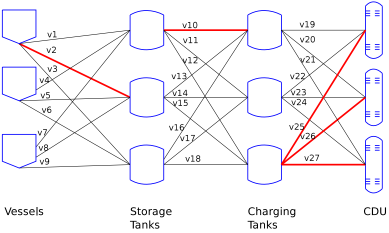
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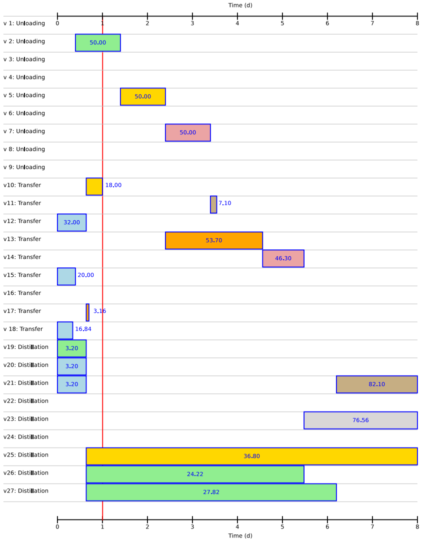
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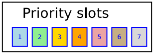
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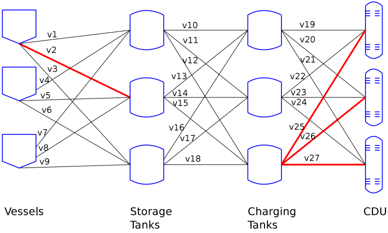
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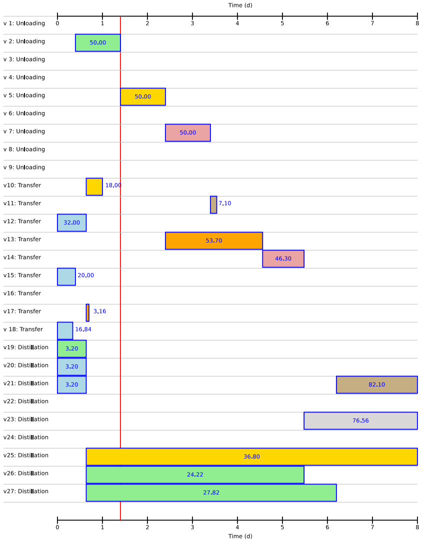
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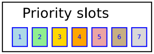
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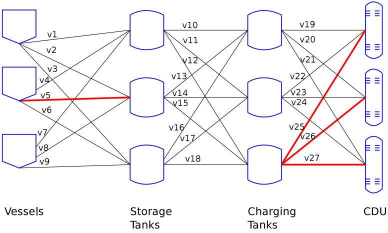
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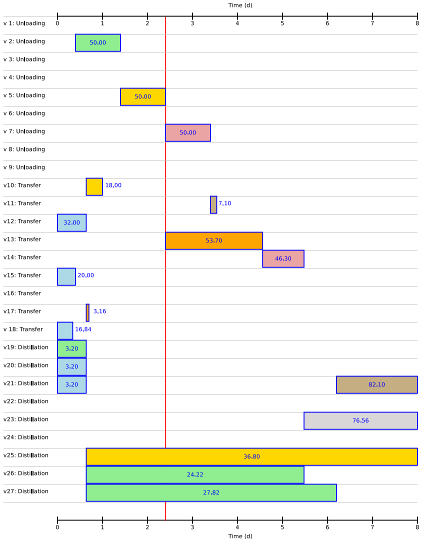
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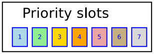
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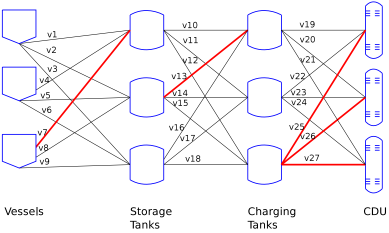
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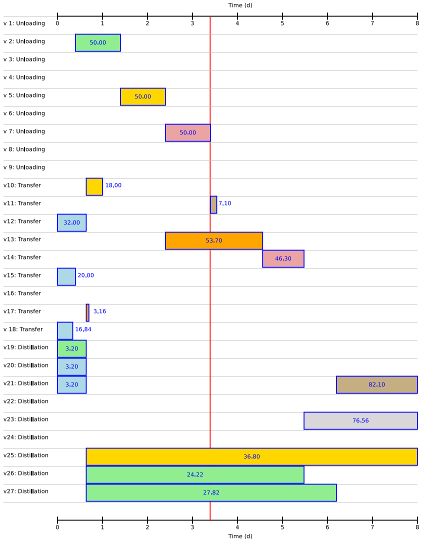
Vessels
Storage Tanks
Charging Tanks
CDU


Time (d)
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v 18: Transfer
16.84
v19: Distillation
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v21: Distillation
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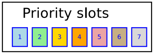
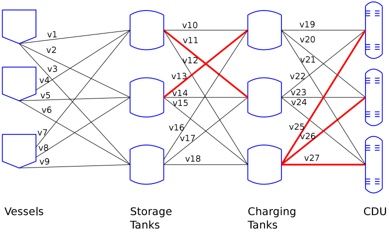
v27: Distillation
27.82
0 1 2 3 4 5 6 7 8
Time (d)


Priority slots
1 2 3 4 5 6 7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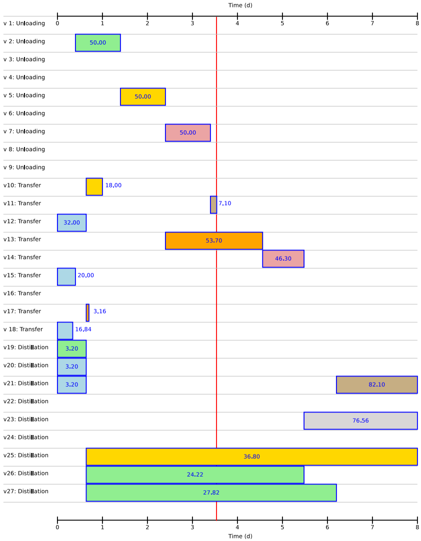
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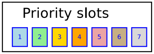
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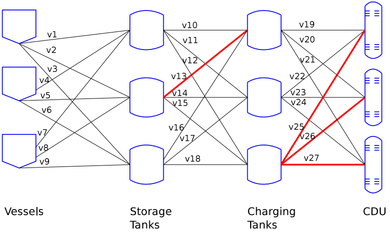
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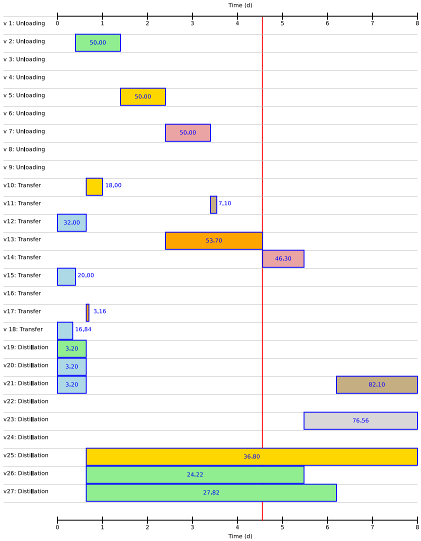
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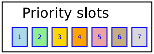
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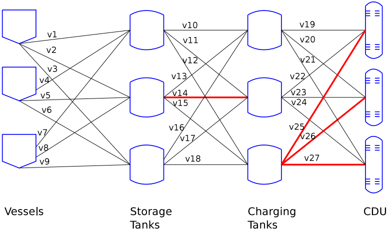
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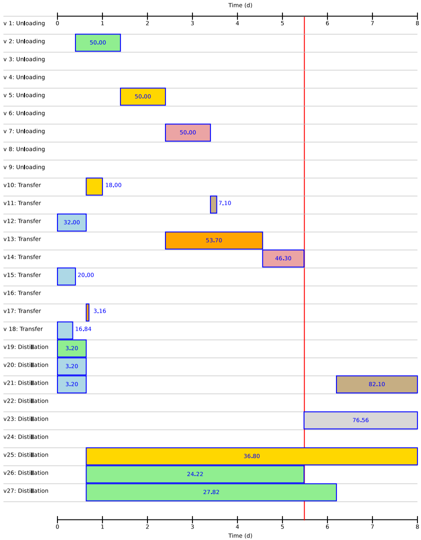
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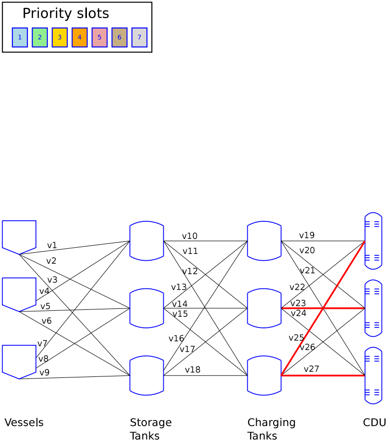
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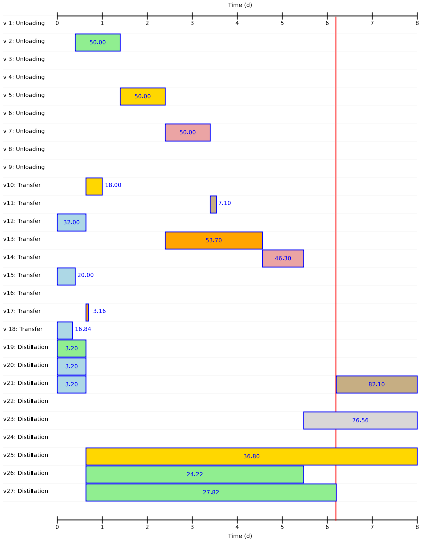
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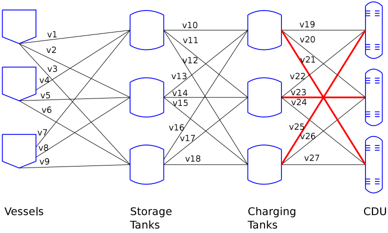
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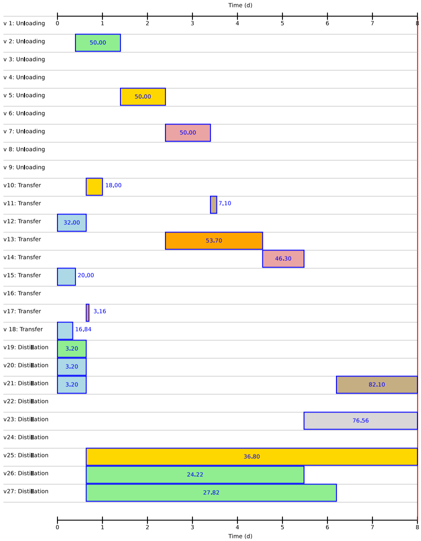
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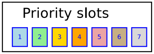
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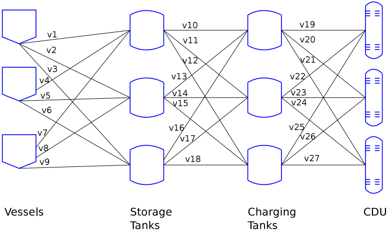
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Partial conclusions and future work

Partial conclusions:

- The **two-stages strategy** (MILP + NLP) gives a solution in a shorter CPU time than the MINLP with global solver.
- Why does this strategy work? Two hypothesis:
  - **Few blending operations** where nonlinearities are active.
  - Nonlinear constraint **has no influence on priorities**.

Future work in this instance:

- Improve the algorithm.
- Decide the number of priority slots.
- Scale up.
- Decomposition.

Scheduling of oil-refinery operations

Felipe Díaz-Alvarado[2], Francisco Trespalacios[1], Ignacio Grossmann[1]

[1] Center for Advanced Process Decision-making. Carnegie Mellon University
[2] Department of Chemical and Biotechnological Engineering. University of Chile

March 2015

felidiaz@ing.uchile.cl

Solution (first obtained and best)

	Global	
# Continuous Vars.:	2311	
# Discrete Vars.:	189	
# Constraints:	5738	
	1st stage (MILP)	**2nd stage (NLP)**
# Removed Constraints:	755	1043
# Constraints:	4983	4695

			First feasible solution			Best solution		
Strategy	First stage	Second stage	CPU Time [s]	Obj. Fun.	Difference with best	CPU Time [s]	Obj. Fun.	Difference with best
MILP + NLP	CPLEX	BARON	3829	60.400	0.71%	3829	60.400	0.71%
MILP + NLP	CPLEX	CONOPT	122	60.400	0.71%	159	60.829	0.00%
MINLP	BARON					(*) 90000	60.793	0.06%

(*) Time limit.

Data for the example

Time horizon: 8 days.

Priority slots: 7

Vessels	Arrival time	Initial crude type	Initial crude vol. [Mbbl]
1	0	1	50
2	0	2	50
3	0	3	50
Storage TK	Capacity [Mbbl]	Initial crude type	Initial crude vol. [Mbbl]
1	100	4	50
2	100	2	20
3	100	2	20
Charging TK	Capacity [Mbbl]	Initial crude type	Initial crude vol. [Mbbl]
1	100	3	20
2	100	4	20
3	100	1	20

Flowrates	LB	UB
Unloading	0	50
Transfer	0	50
Distillation	5	50

Maximum transferable volume [Mbbl]
100

Data for the example

Blend Composition in Charging tanks (vol/vol)				
	Crude 1	Crude 2	Crude 3	Crude 4
Charging TK 1	[0 , 1]	[0 ,0.9]	[0.1, 1]	[0 , 1]
Charging TK 2	[0 , 1]	[0 ,0.9]	[0 , 1]	[0.1, 1]
Charging TK 3	[0.1, 1]	[0 ,0.9]	[0 , 1]	[0 , 1]

Property 1 (sulfur concentration)		Cost [MM$/Mbbl]
Crude 1	0.1	0.05
Crude 2	0.6	0.3
Crude 3	0.2	0.1
Crude 4	0.5	0.25

	Property 1	Revenue [MM$/Mbbl]
Prod. 1	[0.2,0.6]	0.5
Prod. 2	[0.1,0.5]	0.1
Prod. 3	[0.1,0.5]	0.7

Data for the example

Product Yield	Prod. 1	Prod. 2	Prod. 3
Crude 1	0.5	0.3	0.1
Crude 2	0.2	0	0.7
Crude 3	0	0.9	0
Crude 4	0.5	0.25	0.25

Prop. 1 Yield by Product	Prod. 1	Prod. 2	Prod. 3
Crude 1	0.1	0.9	0
Crude 2	0.8	0.1	0.1
Crude 3	0.5	0.3	0.1
Crude 4	0	0.2	0.7

Minimum Product Yield	Prod. 1	Prod. 2	Prod. 3
CDU 1	0	0	0
CDU 2	0	0	0
CDU 3	0	0	0

Demand	Prod. 1	Prod. 2	Prod. 3
CDU 1	[5 ,1000]	[0 ,1000]	[0 ,1000]
CDU 2	[0 ,1000]	[5 ,1000]	[0 ,1000]
CDU 3	[0 ,1000]	[0 ,1000]	[5 ,1000]

Model

General operating constraints

- Time in operations:

$$S_{tw} \geq S_w^{lo} \cdot z_{tw} \quad t \in T, w \in W \tag{1}$$

$$E_{tw} \leq I_w^{up} \cdot z_{tw} \quad t \in T, w \in W \tag{2}$$

$$E_{tw} = S_{tw} + D_{tw} \cdot z_{tw} \quad t \in T, w \in W \tag{3}$$

- Volume and flowrates in operations:

$$V_w^{lo} \cdot z_{tw} \leq V_{tw} \leq V_w^{up} \cdot z_{tw} \quad t \in T, w \in W \tag{4}$$

$$V_{tw} = \sum_{c \in C} \hat{V}_{twc} \quad t \in T, w \in W \tag{5}$$

$$F_w^{lo} \cdot D_{tw} \leq V_{tw} \leq F_w^{up} \cdot D_{tw} \quad t \in T, w \in W \tag{6}$$

$$x_{wk}^{lo} \cdot V_{tw} \leq \sum_{c \in C} x_{ck}^{0} \cdot \hat{V}_{twc} \leq x_{wk}^{up} \cdot V_{tw} \quad t \in T, w \in W, k \in K \tag{7}$$

- No overlapping constraints:

$$z_{tw} + z_{tw'} \leq 1 \qquad t \in T, w \in W, w' \in NO_{w,w'}, w' > w \tag{8}$$

$$E_{t_1 w} + E_{t_1 w'} + \sum_{t \in T}^{t_1 < t < t_2} (D_{tw} + D_{tw'}) \leq S_{t_2 w} + S_{t_2 w'} + H \cdot (1 - z_{t_2 w} - z_{t_2 w'})$$

$$t_1, t_2 \in T, w \in W, w' \in NO_{w,w'}, w' > w \tag{9}$$

Constraints on blending and charging tanks

- Inventory definition:

$$L_{tr} = L_r^0 + \sum_{t' \in T}^{t'<t} \sum_{w \in I_r} V_{t'w} - \sum_{t' \in T}^{t'<t} \sum_{w \in O_r} V_{t'w} \qquad t \in T, r \in R_S \cup R_C \tag{10}$$

$$\hat{L}_{trc} = \hat{L}_{rc}^0 + \sum_{t' \in T}^{t'<t} \sum_{w \in I_r} \hat{V}_{t'wc} - \sum_{t' \in T}^{t'<t} \sum_{w \in O_r} \hat{V}_{t'wc} \qquad t \in T, r \in R_S \cup R_C, c \in C \tag{11}$$

$$L_{tr} = \sum_{c \in C} \hat{L}_{trc} \qquad t \in T, r \in R_S \cup R_C \tag{12}$$

- Blending constraints:

$$\hat{V}_{twc} \cdot L_{tr} = V_{tw} \cdot \hat{L}_{trc} \qquad t \in T, r \in R_S \cup R_C, w \in O_r, c \in C \tag{13}$$

$$CD^{lo}_{rc} \cdot V_{tw} \leq \hat{V}_{twc} \leq CD^{up}_{rc} \cdot V_{tw} \qquad t \in T, r \in R_C, w \in W_D, c \in C \tag{14}$$

- Inventory bounds:

$$L^{lo}_r \leq L_{tr} \leq L^{up}_r \qquad t \in T, r \in R_S \cup R_C \tag{15}$$

$$L^{lo}_r \leq L^0_r + \sum_{t \in T} \sum_{w \in I_r} V_{tw} - \sum_{t \in T} \sum_{w \in O_r} V_{tw} \leq L^{up}_r \qquad r \in R_S \cup R_C \tag{16}$$

Constraints on CDUs

- Limit the number of discharge operations to CDUs:

$$N_r^{lo} \leq \sum_{t \in T} \sum_{w \in I_r} z_{tw} \leq N_r^{up} \qquad r \in R_D \tag{17}$$

- Ensure constant operation:

$$\sum_{t \in T} \sum_{w \in I_r} D_{tw} = H \qquad r \in R_D \tag{18}$$

- Product yield and properties in products:

$$VP_{twp} = \sum_{c \in C} PY^0_{wcp} \cdot \hat{V}_{twc} \qquad t \in T, w \in W_D, p \in P \tag{19}$$

$$\hat{V}P_{twpk} = \sum_{c \in C} PP^0_{wcpk} \cdot PY^0_{wcp} \cdot \hat{V}_{twc} \qquad t \in T, w \in W_D, p \in P, k \in K \tag{20}$$

- Limits on demands, product demands, and specifications in products:

$$DD_r^{lo} \leq \sum_{t \in T} \sum_{w \in I_r} V_{tw} \leq DD_r^{up} \qquad r \in R_D \tag{21}$$

$$PY_{rp}^{lo} \cdot V_{tw} \leq VP_{twp} \qquad t \in T, r \in R_D, w \in I_r, p \in P \tag{22}$$

$$PP_{wpk}^{lo} \cdot VP_{twp} \leq \hat{V}P_{twpk} \leq PP_{wpk}^{up} \cdot VP_{twp} \qquad t \in T, w \in W_D, p \in P, k \in K \tag{23}$$

$$PD_{rp}^{lo} \leq \sum_{t \in T} \sum_{w \in I_r} VP_{twp} \leq PD_{rp}^{up} \qquad p \in P, r \in R_D \tag{24}$$

- Complete unloading of Vessels:

$$\sum_{t \in T} \sum_{w \in O_r} V_{tw} = L_r^0 \qquad r \in R_V \tag{25}$$